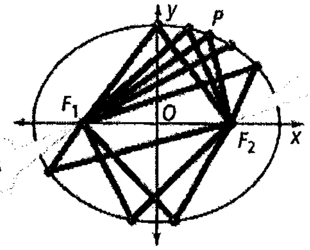
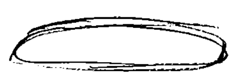
**Honors Algebra 2 Notes** Name: ____________________

**Conic Sections – Ellipses (Day 1)** Date: ______________ Pd: ____

## Definition of an Ellipse:

The set of all points in a plane such that the sum of the distances from two given points in the plane, called the foci, is constant.



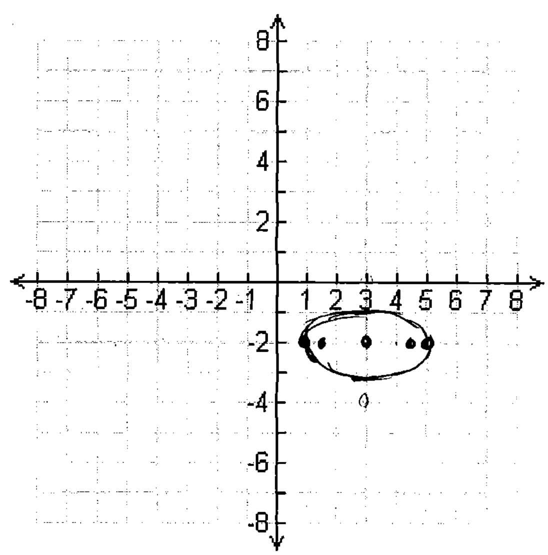

| **Standard form of Equation** | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ | $\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$ |
|---|---|---|
| **Center** | (h, k) | (h, k) |
| **Direction of Major Axis** | horizontal | vertical |
| **Foci** | (h+c, k) + (h-c, k) where $c^2=a^2-b^2$ | (h, k+c) + (h, k-c) where $c^2=a^2-b^2$ |
| **Length of Major Axis** | 2a units | 2a units |
| **Length of Minor Axis** | 2b units | 2b units |
| **Sketch** |  |  |

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

1. $\frac{(x-3)^2}{4}+\frac{(y+2)^2}{1}=1$

$a=2$ $b=1$

$c^2=2^2-1^2$
$c^2=4-1$
$c^2=3$
$c=\sqrt{3}$




Center: (3, -2)

Direction of major axis: horizontal

Foci: $(3\pm\sqrt{3}, -2)$ = (4.7, -2) + (1.3, -2)

Length of major axis: 2(2) = 4

Length of minor axis: 2(1) = 2

① plot center
② count - major axis
③ count - minor axis

Find the coordinates of the center and foci and the lengths of the major and minor axes for the ellipse with the given equation. Then graph the ellipse.

2. $\frac{9x^2}{36} + \frac{4y^2}{36} = \frac{36}{36}$ $a > b!$

$\frac{x^2}{4} + \frac{y^2}{9} = 1 \rightarrow \frac{y^2}{9} + \frac{x^2}{4} = 1$

$a = 3 \quad b = 2$

$c^2 = 3^2 - 2^2$

$c^2 = 9 - 4$

$c^2 = 5$

$c = \sqrt{5}$

$c \approx 2.23$

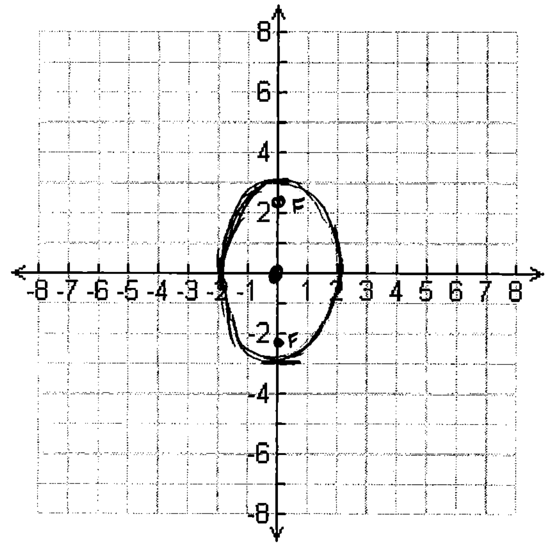
Center: $(0,0)$

Direction of major axis: vertical

Foci: $(0, 0 \pm \sqrt{5}) = (0, \sqrt{5})$ + $(0, -\sqrt{5})$

Length of major axis: $2(3) = 6$

Length of minor axis: $2(2) = 4$



try on your own!

3. $\frac{25x^2}{225} + \frac{9y^2}{225} = \frac{225}{225}$

$\frac{x^2}{9} + \frac{y^2}{25} = 1 \rightarrow \frac{y^2}{25} + \frac{x^2}{9} = 1$

$a = 5 \quad b = 3$

$c^2 = 5^2 - 3^2$

$c^2 = 25 - 9$

$c^2 = 16$

$c = 4$

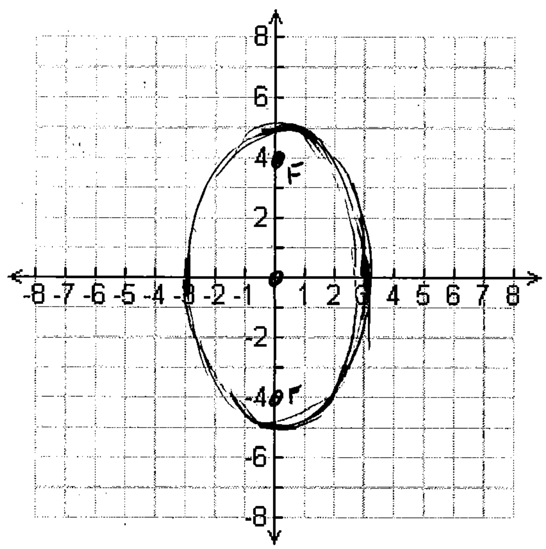
Center: $(0,0)$

Direction of major axis: Vertical

Foci: $(0, \pm 4) = (0, 4)$ + $(0, -4)$

Length of major axis: $2(5) = 10$

Length of minor axis: $2(3) = 6$



**Algebra 2 Notes** Name: ____________________

**Conic Sections – Ellipses (Day 2)** Date: ______________ Pd: ____

1. Write the equation in standard form then find the center and foci.

$$x^2+4y^2+4x-24y+24=0$$

$x^2+4x+\boxed{\phantom{0}}+4y^2-24y+\boxed{\phantom{0}}=-24+\boxed{\phantom{0}}+\boxed{\phantom{0}}$

$x^2+4x+(2)^2+4(y^2-6y+(-3)^2)=-24+\boxed{(2)^2}+\boxed{4(-3)^2}$

$(x+2)^2+4(y-3)^2=-24+4+4(9)$

$\frac{(x+2)^2}{16}+\frac{4(y-3)^2}{16}=\frac{16}{16}$ $a=4$ $b=2$ $c^2=4^2-2^2$

$c^2=16-4$

$\sqrt{c^2}=\sqrt{12}<^4_3$

$c=2\sqrt{3}$

$\boxed{\frac{(x+2)^2}{16}+\frac{(y-3)^2}{4}=1}$ major axis horizontal

Center: $(-2,3)$
Foci: $(-2\pm2\sqrt{3},3)$

2. Write the equation in standard form then find the center and foci.

$$16x^2+25y^2+32x-150y=159$$

$16x^2+32x+\boxed{\phantom{0}}+25y^2-150y+\boxed{\phantom{0}}=159+\boxed{\phantom{0}}+\boxed{\phantom{0}}$

$16(x^2+2x+\boxed{(1)^2})+25(y^2-6y+\boxed{(-3)^2})=159+\boxed{16(1)^2}+\boxed{25(-3)^2}$

$\frac{16(x+1)^2}{400}+\frac{25(y-3)^2}{400}=\frac{400}{400}$ $a=5$ $b=4$ $c^2=5^2-4^2$

$c^2=25-16$

$c^2=9$

$c=3$

major axis horizontal

$\boxed{\frac{(x+1)^2}{25}+\frac{(y-3)^2}{16}=1}$

center: $(-1,3)$
foci: $(-1\pm3,3)=(2,3)$ & $(-4,3)$

Write the equation of an ellipse with the following characteristics:

3. Endpoints of major axis at (0, 6) and (0, -6), endpoints of minor axis at (-3, 0) and (3, 0).

major axis: $(0,-6)$ $(0,6)$ $a=6$ $a^2=36$

minor axis: $(0,-3)$ $(0,3)$ $b=3$ $b^2=9$

center: $\left(\frac{0+6}{2},\frac{-6+6}{2}\right)=(0,0)$

$\boxed{\frac{x^2}{9}+\frac{y^2}{36}=1}$

4. Major axis 12 units long and parallel to the x-axis, minor axis 4 units long, center at (0, 0).

major axis = 12 units $2a=12$ $a=6$ $a^2=36$

minor axis = 4 units $2b=4$ $b=2$ $b^2=4$

parallel to x-axis = horizontal

$\boxed{\frac{x^2}{36}+\frac{y^2}{4}=1}$

5. Endpoints of major axis at (0, 12) and (0, -12) and foci at $(0, \overset{0+c}{\sqrt{23}})$ and $(0, \overset{0-c}{-\sqrt{23}})$.

major axis: $(0,-12)$ $(0,12)$

center: $\left(\frac{0+0}{2}, \frac{-12+12}{2}\right) = (0,0)$

$a = 12$ $\quad a^2 = 144$

$b = 11$ $\quad b^2 = 121$

Foci $= (h \pm c, k)$

$(0 \pm \sqrt{23}, 0)$

$c = \sqrt{23}$

$c^2 = 23$

$$\begin{aligned} 23 &= 144 - b^2 \\ -23 &\quad -23 \\ \hline 0 &= 121 - b^2 \\ b^2 &= 121 \\ b &= 11 \end{aligned}$$

$$\boxed{\frac{x^2}{121} + \frac{y^2}{144} = 1}$$

6. Major axis 16 units long, center at (0, 0) and foci at $(0, 2\sqrt{15})$ and $(0, -2\sqrt{15})$.

$a = 8$ $\quad a^2 = 64$

$c = 2\sqrt{15}$ $\quad c^2 = (2\sqrt{15})^2 = 4 \cdot 15 = 60$

$$\begin{aligned} 60 &= 64 - b^2 \\ -60 &\quad -60 \\ \hline 0 &= 4 - b^2 \\ b^2 &= 4 \\ b &= 2 \end{aligned}$$

$$\boxed{\frac{x^2}{4} + \frac{y^2}{64} = 1}$$

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**<u>Practice Problems:</u> Work on a separate sheet of paper and show ALL steps!!**

Write each equation in standard form. Identify the center, foci and lengths of the major and minor axes.

1. $x^2 + 4y^2 + 8x - 64y = -128$ 2. $x^2 + 4x + 2y^2 + 16y + 32 = 0$

Write an equation for the ellipse that satisfies each set of conditions.

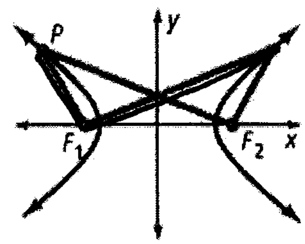
3. Endpoints of major axis at (2, 6) and (8, 6), endpoints of minor axis at (5, 4) and (5, 8).
4. Endpoints of major axis at (-6, 0) and (6, 0) and foci at $(-\sqrt{32}, 0)$ and $(\sqrt{32}, 0)$.
5. Major axis 10 units long, minor axis 6 units long and parallel to x-axis, center at (2, -4).
6. Endpoints of minor axis at (0, 2) and (0, -2) and foci at (-4, 0) and (4, 0).

**Algebra 2 Notes** Name: ____________________

**Conic Sections – Hyperbolas (8.5)** Date: ______________ Pd: ____

## Hyperbolas!

1. A hyperbola is the set of all points in a plane such that the absolute value of the difference of the distances from any point on the hyperbola to two fixed points $F_1$ and $F_2$, the foci, is a constant, $k$.



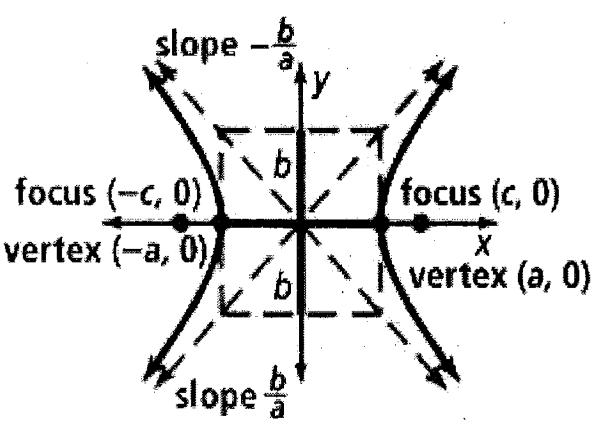

   a. The hyperbola has two parts or branches.
   b. The midpoint of the segment connecting the foci of the hyperbola is the center of the hyperbola.
   c. The point on each branch of the hyperbola that is nearest the center is a vertex.

2. A line that a graph approaches but never crosses is called an asymptote.
   a. As the hyperbola recedes from the center, the branches approach the asymptote.

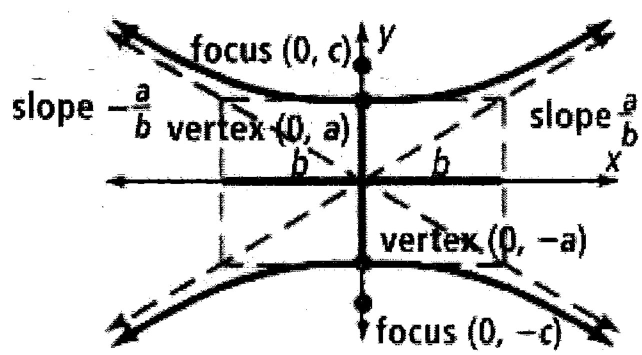
3. A hyperbola has many similarities to an ellipse.
   a. There are 2 axes of symmetry.
   b. The transverse axis is a segment with length $2a$ units whose endpoints are the vertices of the hyperbola.
   c. The conjugate axis is a segment with length $2b$ units that is perpendicular to the transverse axis at the center.

## Hyperbolas with Center (0, 0)

### Horizontal Hyperbola




### Vertical Hyperbola



$+x^2 - y^2$ so $+, a =$ horizontal $\quad +y^2 - x^2$, so trans. axis vertical

| Standard form of Equation | $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ | $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ |
|---|---|---|
| **Center** | $(h, k)$ | $(h, k)$ |
| **Direction of Transverse Axis** | horizontal | vertical |
| **Foci** | $(h \pm c, k)$ where $c^2 = a^2 + b^2$ | $(h, k \pm c)$ where $c^2 = a^2 + b^2$ |
| **Vertices** | $(h \pm a, k)$ | $(h, k \pm a)$ |
| **Length of Transverse Axis** | $2a$ | $2a$ |
| **Length of Conjugate Axis** | $2b$ | $2b$ |
| **Equations of Asymptotes** | $y - k = \pm \frac{b}{a}(x - h)$ | $y - k = \pm \frac{a}{b}(x - h)$ |

*opens left/right $\quad$ *opens up/down

Find the coordinates of the center, vertices, foci and the equations of the asymptotes for the hyperbola with the given equation. Then graph the hyperbola.

1. $\frac{x^2}{49} - \frac{y^2}{16} = 1 \qquad a^2 = 49 \quad a = 7 \qquad b^2 = 16 \quad b = 4$

Center: $(0, 0)$

Direction of transverse axis: horizontal

Foci: $(0 \pm \sqrt{65}, 0) \approx (\pm 8.06, 0)$

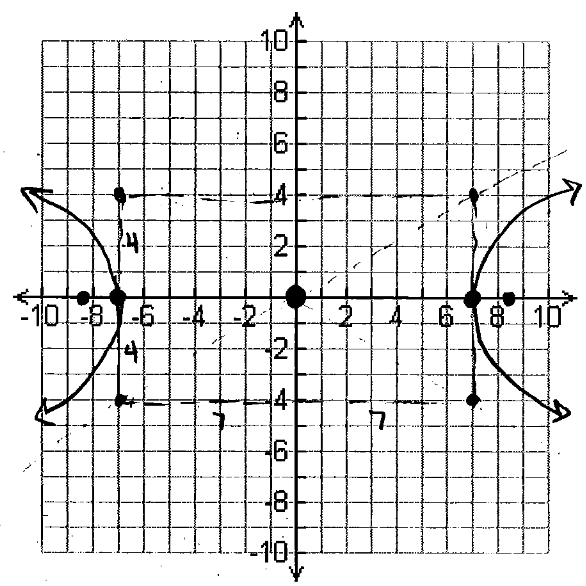
Vertices: $(0 \pm 7, 0) = (\pm 7, 0)$

Length of transverse axis: $2 \cdot 7 = 14$

Length of conjugate axis: $2 \cdot 4 = 8$

Equations of Asymptotes: $y - 0 = \pm \frac{4}{7}(x - 0)$

$y = \pm \frac{4}{7}x$




$c^2 = a^2 + b^2$

$c^2 = 49 + 9$

$c^2 = 65 \quad c = \sqrt{65}$

to graph

1. plot center
2. plot vertices
3. verify trans. axis
4. plot conj. axis
5. asymptotes → graph the lines
6. draw curves (don't cross asymptotes!)

7. foci?

$x^2$ is negative so trans. axis is

2. $\frac{(\overset{k}{y-3})^2}{1} - \frac{(\overset{h}{x+2})^2}{9} = 1$

$a^2=1 \quad a=1$

$b^2=9 \quad b=3$

Center: $(-2,3)$ ← careful!

Direction of transverse axis: vertical

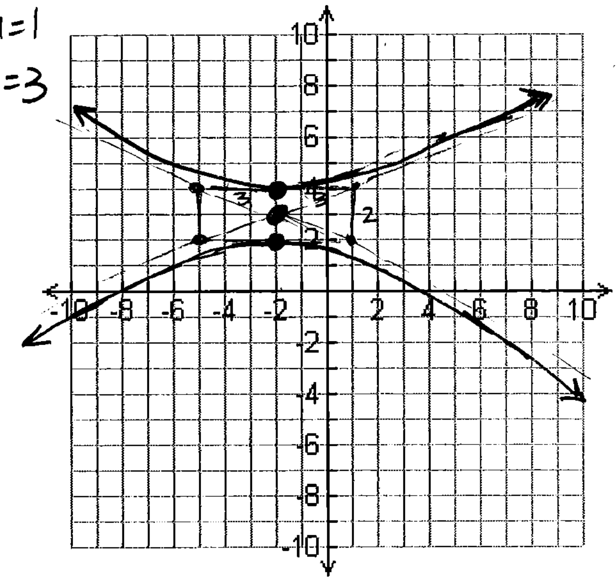
$(h,k\pm c)$ Foci: $(-2, 3\pm\sqrt{10})$

$(h,k\pm a)$ Vertices: $(-2, 3\pm 1) = (-2,4)$ & $(-2,2)$

$2a$ Length of transverse axis: $2(1) = 2$

$2b$ Length of conjugate axis: $2(3) = 6$

$y-k = \pm\frac{a}{b}(x-h)$ Equations of Asymptotes: $y-3 = \pm\frac{1}{3}(x+2)$




$c^2 = 1+9$

$c^2 = 10 \quad c = \sqrt{10}$

3. $x^2 - 6x - 4y^2 + 32y - 71 = 0$

$x^2 - 6x + \boxed{(-3)^2} - 4(y^2 - 8y + \boxed{(-4)^2}) = 71 + \boxed{(-3)^2} + -4\boxed{(-4)^2}$

$(x-3)(x-3) - 4(y-4)(y-4) = 71 + 9 + -4\underset{64}{(16)}$

$\frac{(x-3)^2}{16} - \frac{4(y-4)^2}{16} = \frac{16}{16}$

$$\boxed{\frac{(x-3)^2}{16} - \frac{(y-4)^2}{4} = 1}$$

$a^2=16$

$a=4$

$b^2=4$

$b=2$

Center: $(3,4)$

Direction of transverse axis: horizontal

$(h\pm c, k)$ Foci: $(3\pm\sqrt{20}, 4)$

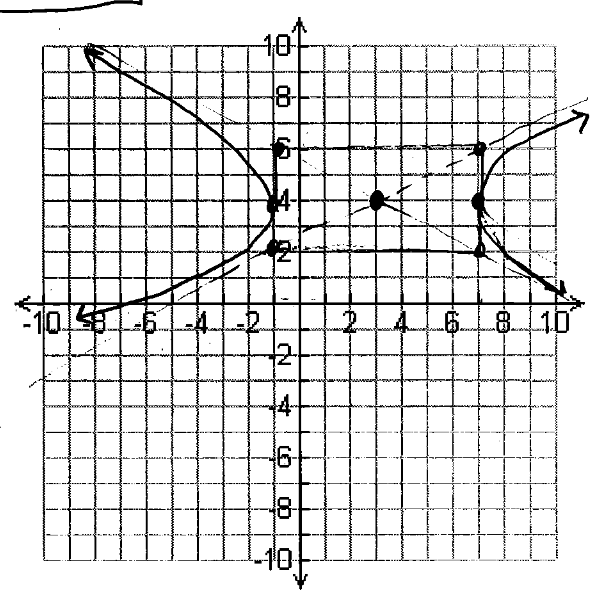
$(h\pm a, k)$ Vertices: $(3\pm 4, 4) = (7,4)$ & $(-1,4)$

$2a$ Length of transverse axis: $2\cdot 4 = 8$

$2b$ Length of conjugate axis: $2\cdot 2 = 4$

$y-k = \pm\frac{b}{a}(x-h)$ Equations of Asymptotes: $y-4 = \pm\frac{2}{4}(x-3)$

$y-4 = \pm\frac{1}{2}(x-3)$

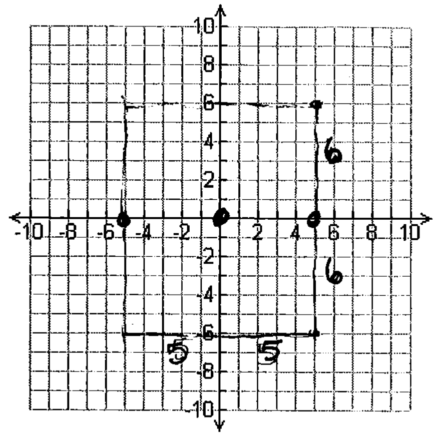



$c^2 = 16+14$

$c^2 = 20 \quad c = \sqrt{20}$

Write an equation for the hyperbola that satisfies each set of conditions.

4. vertices (–5, 0) and (5, 0), conjugate axis of length 12




① sketch

② $a=5 \quad b=6$

③ center? $(\overset{h}{0},\overset{k}{0})$

$$\frac{x^2}{5^2} - \frac{y^2}{6^2} = 1$$

$$\boxed{\frac{x^2}{25} - \frac{y^2}{36} = 1}$$

~~5~~. vertices (–3, 0) and (3, 0), foci $(\pm 5, 0)$

remove

remove ~~6~~. vertices (9, –3) and (–5, –3), foci $\left(2 \pm \sqrt{53}, -3\right)$

5. $4x^2 - 25y^2 - 8x - 96 = 0$

$$4x^2 - 8x + \square - 25y^2 = 96$$

$$4(x^2 - 2x + \boxed{(-1)^2}) - 25y^2 = 96 + 4(\boxed{(-1)^2})$$

$$4(x-1)(x-1) - 25y^2 = 96 + 4$$

$$\frac{4(x-1)^2}{100} - \frac{25y^2}{100} = \frac{100}{100}$$

$$\boxed{\frac{(x-1)^2}{25} - \frac{y^2}{4} = 1}$$

**Algebra 2 Notes** Name: ____________________

**Conic Sections (8.6)** Date: ______________ Pd: ____

# Conic Sections – Putting it all together!

| Parabola | Circle | Ellipse | Hyperbola |
|---|---|---|---|
| $y=a(x-h)^2+k$<br>or<br>$x=a(y-k)^2+h$<br>* only one variable is squared (the other is a single+) | $(x-h)^2+(y-k)^2=r^2$<br>* coefficients for $x^2$ and $y^2$ are the same<br>* no fractions<br>* not necessarily equal to 1. | $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$<br>or<br>$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$<br>* $x^2$ and $y^2$ coefficients are different<br>* equal to 1<br>* both $x^2$ and $y^2$ are positive. | $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$<br>or<br>$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$<br>* $x^2$ or $y^2$ is negative<br>* equal to 1. |

Write each equation in standard form. State whether the graph of the equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*. Then graph the equation.

1. $\frac{x^2}{25}-\frac{25y^2}{25}=\frac{25}{25}$

$\frac{x^2}{25}-\frac{y^2}{1}=1$

center at (0,0)

horizontal

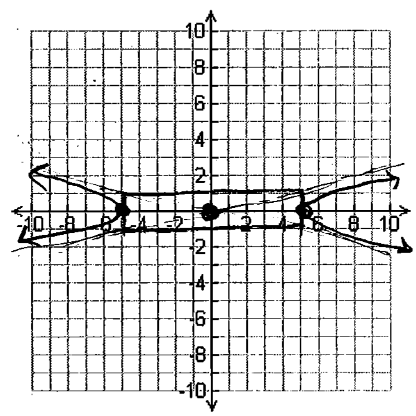
$a=5 \quad b=1$

$c^2=5^2+1^2$

$c^2=25+1$

$c^2=26 \quad c=\sqrt{26}$

$\boxed{\text{hyperbola}}$



2. $\frac{9x^2}{36}+\frac{4y^2}{36}=\frac{36}{36}$

$\frac{x^2}{4}+\frac{y^2}{9}=1$

$\boxed{\text{ellipse}}$

major axis vertical

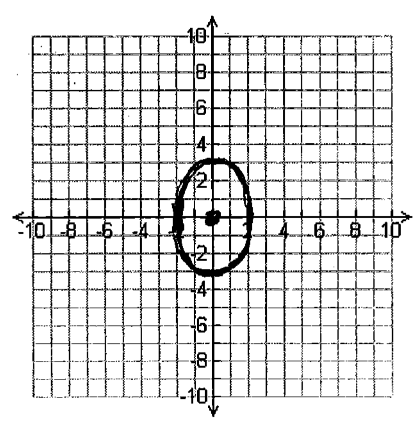
center at (0,0)

$a=3 \quad b=2$

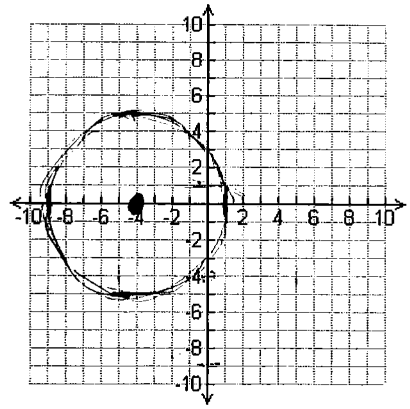
$c^2=3^2-2^2$

$c^2=9-4$

$c^2=5 \quad c=\sqrt{5}$



3. $x^2+8x+y^2=9$



$x^2+8x+\boxed{4^2}+y^2=9+4^2$

$(x+4)(x+4)+y^2=9+16$

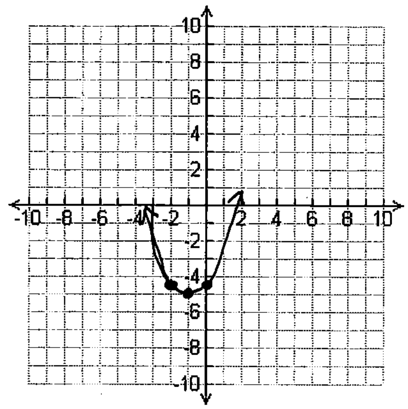
$(x+4)^2+y^2=25$

$\boxed{\text{circle}}$

center at $(-4,0)$

radius $=5$

4. $x^2+2x-4=y$



$\boxed{\text{parabola}}$ → $y=$ (y is a singlet!)

$x^2+2x+1^2=y+4+1^2$

$(x+1)(x+1)=y+5$

$(x+1)^2-5=y$

$a=1$

$LLR=\frac{1}{1}$

vertex: $(-1,-5)$

Without writing the equation in standard form, state whether the graph of each equation is a *parabola*, *circle*, *ellipse*, or *hyperbola*.

different coefficients

5. $16x^2+1y^2=16$

ellipse $(x^2+y^2)$

6. $y=x^2+2x$

parabola

($x^2$, y singlet)

7. $4y^2-25x^2=100$

hyperbola

8. $5x^2+5y^2=25$

circle

(same coefficient)

9. $25y^2-50y+4x^2=75$

ellipse

($x^2+y^2$, different coefficient)

10. $(x+3)^2+(y-1)^2=4$

circle

11. $x^2-6y^2+9=0$

hyperbola

12. $6=y^2+5y-x$

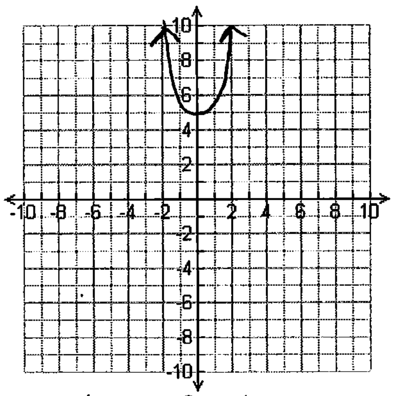
singlet

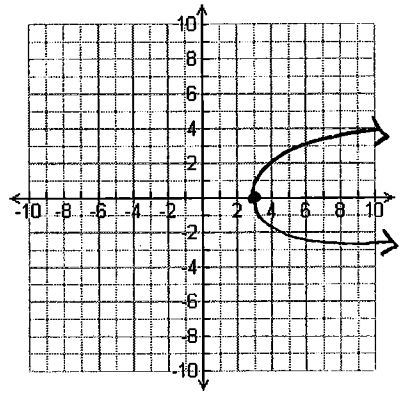
parabola

**Algebra 2** Name: ____________________

# Conic Sections Review!!!

Parabolas




Equations: $y = a(x-h)^2 + k$ $x = a(y-k)^2 + h$

Key Parts: $(h, k)$ = vertex a = use in focus & directrix

Information provided: equation, focus, directrix, A.O.S.

Information NOT provided: LLR

Other things to remember: y = ↑ or ↓ (+a, −a) x = ⊃ or ⊂ (−a, +a) make a table of values to graph 5 points!

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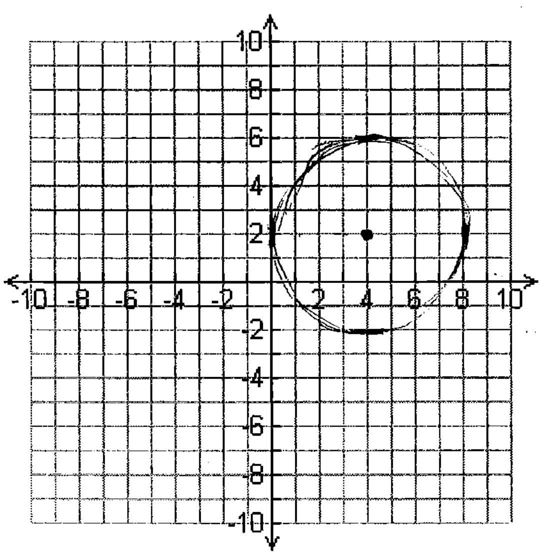
Circles

Equation: $(x-h)^2 + (y-k)^2 = r^2$




Key Parts: center = $(h, k)$
radius = $r$

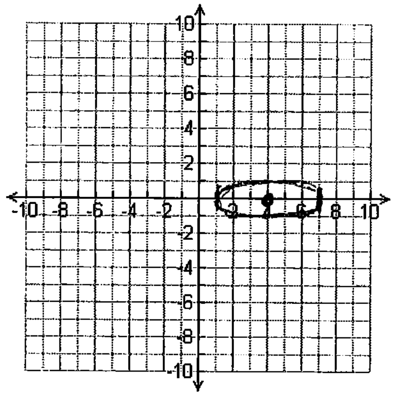
Information provided: all!

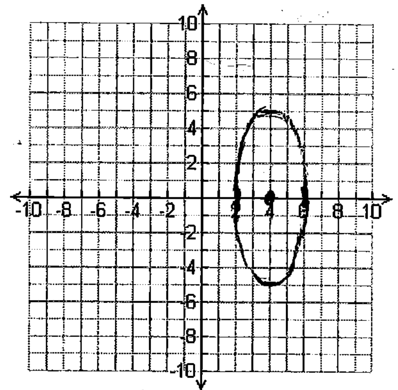
Information NOT provided: none

Other things to remember:

- If given 2 points on the circle
  1. midpoint to find center
  2. distance to find radius (between center and one other point)
- If given center & point, just use distance formula.

## Ellipse





Equations: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$

Key Parts: center : $(h, k)$
$a$ = major axis "radius" $b$ = minor axis "radius"

Information provided: focus, $c^2 = a^2 - b^2$, equations

Information NOT provided: major axis = $2a$ minor axis = $2b$ $a = \sqrt{\ }$ of bigger denominator $b = \sqrt{\ }$ of smaller denominator

Other things to remember: ellipse is $x^2 + y^2$ but different coefficients.

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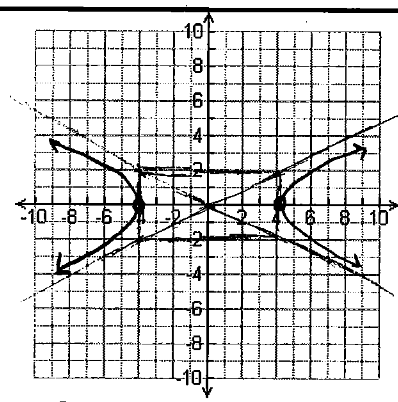
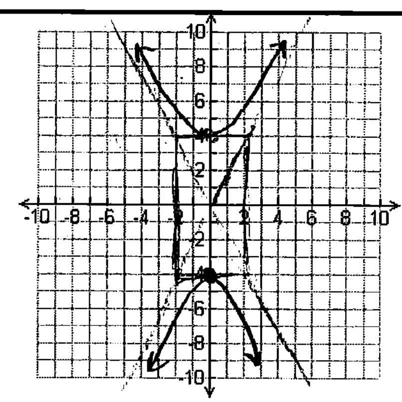
## Hyperbolas





Equations: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

Key Parts: center: $(h, k)$ transverse axis: $2a$ conjugate axis: $2b$ vertices: $(h \pm a, k)$ asymptotes!

Information provided: equation, center, foci, $c^2 = a^2 + b^2$

Information NOT provided: Trans. axis, conj. axis, vertices, asymptotes.

Other things to remember: the − runs the show!
to graph ① box ② asymptotes ③ curves